2.1 The Nature of Sets

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Sets

A **set** is a well-defined collection of objects.

By *well-defined* we mean that given any object, we can definitely decide whether it is or is not in the set.

Each object in a set is called an **element** or a **member** of the set.

Ex.) Write the set of months of the year that begin with the letter M.

M = {March, May}

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Three Methods Commonly Used to Designate a Set:

- Roster Form
  The elements of the set are listed between braces, with commas between the elements.
- Description
  This uses a short statement to describe the set.
- Set-Builder Notation
  This method uses a variable, braces, and a vertical bar | that is read as "such that."

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Sets are generally named with a capital letter

- The Set of Natural Numbers (Counting Numbers) is listed:
  N = {1, 2, 3, 4 ...}
- The Integers:
  I = {...-3, -2, -1, 0, 1, 2, 3...}

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Ex.) Use the roster method to do the following:

(a) Write the set of natural numbers less than 6.

{1, 2, 3, 4, 5}

(b) Write the set of natural numbers greater than 4.

{5, 6, 7, 8, ...}

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Set Notation

The symbol $\in$ is used to show that an object is a member or element of a set.
For example, let set *A = {2, 3, 5, 7, 11}*.

Since 2 is a member of set A, it can be written as 2 $\in$ {2, 3, 5, 7, 11} or 2 $\in$ *A*

Likewise, 5 $\in$ {2, 3, 5, 7, 11} or 5 $\in$ *A*

When an object is not a member of a set, the symbol $\notin$ is used.

Because 4 is not an element of set *A, this fact is written as* 4 $\notin$ {2, 3, 5, 7, 11} or 4 $\notin$ *A*

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Ex.) Decide whether each statement is true or false.

(a) Oregon $\in$ *A, where A is the set of states west of the Mississippi River.*

True

(b) 27 $\in$ {1, 5, 9, 13, 17, . . .} 21, 25, 29, ...

False

(c) z $\notin$ {v, w, x, y, z}

False

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Ex.) Use the descriptive method to describe the set *E containing 2, 4, 6, 8, ...*

The set E is the set of even natural numbers.

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Set-Builder Notation

Such that

S = { x | condition(s) }

Set S

is

The Set of

All elements x

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Ex.) Use set-builder notation to designate each set, then write how your answer would be read aloud.

(a) The set R *contains the elements 2, 4, and 6.*

$R = \{x \mid x \in E \text{ and } x < 7\}$

(b) The set *W contains the elements red, yellow, and blue.*

$W = \{x \mid x \text{ is a primary color}\}$

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Ex.) Designate the set *S* with elements 32, 33, 34, 35, … using

(a) The roster method.

$\{32, 33, 34, 35, \ldots\}$

(b) The descriptive method.

The set S is the set of natural numbers greater than 31

(c) Set-builder notation.

$\{x \mid x \in N \text{ and } x > 31\}$

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Ex.) Using the roster method, write the set containing all even natural numbers between 99 and 201.

$\{100, 102, \ldots, 198, 200\}$

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Finite and Infinite Sets

If a set has no elements or a specific natural number of elements, then it is called a **finite set.** A set that is not a finite set is called an **infinite set.**

The set {p, q, r, s} is called a **finite set** since it has four members: p, q, r, and s.

The set {10, 20, 30, . . .} is called an infinite set since it has an unlimited number of elements: the natural numbers that are multiples of 10.

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Ex.) Classify each set as finite or infinite.

(a) $\{x \mid x \in N \text{ and } x < 100\}$ Finite

(b) Set *R* is the set of letters used to make Roman numerals. Finite

(c) {100, 102, 104, 106, . . .} Infinite

(d) Set *M* is the set of people in your immediate family. Finite

(e) Set *S* is the set of songs that can be written. Infinite

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Empty Set or Null Set

A set with no elements is called an *empty set* or **null set.** The symbols used to represent the null set are { } or ∅.

Ex.) Which of the following sets are empty?

(a) The set of woolly mammoth fossils in museums. Not empty

(b) {*x* | *x* is a living woolly mammoth} Empty

(c) {∅} Not empty

(d) {*x* | *x* is a natural number between 1 and 2} Empty

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Cardinal Number

The **cardinal number** of a finite set is the number of elements in the set. For a set *A* the symbol for the cardinality is $n(A)$, which is read as "*n* of *A*."

For example, the set $R = \{2, 4, 6, 8, 10\}$ has a cardinal number of 5 since it has 5 elements. This could also be stated by saying the **cardinality** of set *R* is 5.

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Ex.) Find the cardinal number of each set.

(a) $A = \{5, 10, 15, 20, 25, 30\}$ $n(A)=6$

(b) $B = \{10, 12, 14, \ldots, 28, 30\}$ $n(B)=11$

(c) $C = \{16\}$ $n(C)=1$

(d) ∅

$n(\emptyset)=0$

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Equal and Equivalent Sets

Two sets *A* and *B* are **equal** (written $A = B$) if they have exactly the same members or elements. Two finite sets *A* and *B* are said to be **equivalent** (written $A \cong B$) if they have the same number of elements: that is, $n(A) = n(B)$.

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Ex.) State whether each pair of sets is equal, equivalent, or neither.

(a) {p, q, r, s}; {a, b, c, d} Equivalent

(b) {8, 10, 12}; {12, 8, 10} Equal and Equivalent

(c) {213}; {2, 1, 3} Neither

(d) {1, 2, 10, 20}; {2, 1, 20, 11} Equivalent

(e) {even natural numbers less than 10}; {2, 4, 6, 8} Equal and Equivalent

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One-to-One Correspondence

Two sets have a **one-to-one correspondence** of elements if each element in the first set can be paired with exactly one element of the second set and each element of the second set can be paired with exactly one element of the first set.

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Ex.) Show that ...

(a) the sets {8,16, 24, 32} and {s, t, u, v} have a one-to-one correspondence and

(b) the sets {x, y, z} and {5, 10} do not have a one-to-one correspondence.

(a) {8, 16, 24, 32}

{s, t, u, v}

(b) {x, y, z}

{5, 10}

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Correspondence vs Equivalence

Two sets are

- Equivalent if you can put their elements in one-to-one correspondence.
- Not equivalent if you cannot put their elements in one-to-one correspondence.

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Take out your student handbook and write down the homework assignment:

pg. 50 - 51 #7 - 89 every other odd

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